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Sensitivity Analysis[1]

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Sensitivity analysis measures the impact on project outcomes of changing one or more key input values about which there is uncertainty. For example, a pessimistic, expected, and optimistic value might be chosen for an uncertain variable. Then an analysis could be performed to see how the outcome changes as each of the three chosen values is considered in turn, with other things held the same.

In engineering economics, sensitivity analysis measures the economic impact resulting from alternative values of uncertain variables that affect the economics of the project. When computing **measures of project worth,** for example, sensitivity analysis shows just how sensitive the economic payoff is to uncertain values of a critical input, such as the **discount rate** or project maintenance costs expected to be incurred over the project's **study period.** Sensitivity analysis reveals how profitable or unprofitable the project might be if input values to the analysis turn out to be different from what is assumed in a single-answer approach to measuring project worth.

Sensitivity analysis can also be performed on different combinations of input values. That is, several variables are altered at once and then a measure of worth is computed. For example, one scenario might include a combination of all pessimistic values, another all expected values, and a third all optimistic values. Note, however, that sensitivity analysis can in fact be misleading [Hillier, 1969] if all pessimistic assumptions or all optimistic assumptions are combined in calculating economic measures. Such combinations of inputs would be unlikely in the real world.

Sensitivity analysis can be performed for any measure of worth. And since it is easy to use and understand, it is widely used in the economic evaluation of government and private-sector projects. Office of Management and Budget [1992] Circular A-94 recommends sensitivity analysis to federal agencies as one technique for treating uncertainty in input variables. And the American Society for Testing and Materials (ASTM) [1994], in its *Standard Guide for Selecting Techniques for Treating Uncertainty and Risk in the Economic Evaluation of Buildings and Buildings Systems,* describes sensitivity analysis for use in government and private-sector applications.

187.1 Sensitivity Analysis Applications

How to use sensitivity analysis in engineering economics is best illustrated with examples of applications. Three applications are discussed. The first two focus on changes in project worth

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as a function of the change in one variable only. The third allows for changes in more than one uncertain variable.

The results of sensitivity analysis can be presented in text, tables, or graphs. The following illustration of sensitivity analysis applied to a programmable control system uses text and a simple table. Subsequent illustrations use graphs. The advantage of using a graph comes from being able to show in one picture the outcome possibilities over a range of input variations for one or several input factors.

Sensitivity Table for Programmable Control System

Consider a decision on whether or not to install a programmable time clock to control heating, ventilating, and air conditioning (HVAC) equipment in a commercial building. The time clock would reduce electricity consumption by turning off that part of the HVAC equipment that is not needed during hours when the building is unoccupied.

Using **net savings** (NS) as the measure of project worth, the time clock is acceptable on economic grounds if its NS is positive—that is, if its **present value** savings exceed present value costs. The control system purchase and maintenance costs are felt to be relatively certain. The savings from energy reductions resulting from the time clock, however, are not certain. They are a function of three factors: the initial price of energy, the rate of change in energy prices over the life cycle of the time clock, and the number of kilowatt hours (kWh) saved. Two of these, the initial price of energy and the number of kWh saved, are relatively certain. But future energy prices are not.

To test the sensitivity of NS to possible energy price changes, three values of energy price change are considered: a low rate of energy price escalation (slowly increasing benefits from energy savings), a moderate rate of escalation (moderately increasing benefits), and a high rate of escalation (rapidly increasing benefits).

Table 187.1 Energy Price Escalation Rates

Energy Price Escalation Rate	Net Savings
Low	\$–15 000
Moderate	20 000
High	50 000

Table 187.1 shows three NS estimates that result from repeating the NS computation for each of the three energy price escalation rates.

To appreciate the significance of these findings, it is helpful to consider what extra information is gained over the conventional single-answer approach, where, say, a single NS estimate of \$20 000 was computed. Table 187.1 shows that the project could return up to \$50 000 in NS if future energy prices escalated at a high rate. On the other hand, it is evident that the project could lose as much as \$15 000. This is considerably less than **breakeven,** where the project would at least pay for itself. It is also \$35 000 less than what was calculated with the single-answer approach. Thus, sensitivity analysis reveals that accepting the time clock could lead to an uneconomic outcome.

There is no explicit measure of the likelihood that any one of the NS outcomes will happen. The analysis simply shows what the outcomes will be under alternative conditions. However, if there is reason to expect energy prices to rise, at least at a moderate rate, then the project very likely will make money, other factors remaining the same. This adds helpful information over the traditional single-answer approach to measures of project worth.

Sensitivity Graph for Gas Heating Systems

Figure 187.1 shows how sensitive NS is to the time over which two competing gas heating systems might be used in a building. The sensitivity graph helps you decide which system to choose on economic grounds.

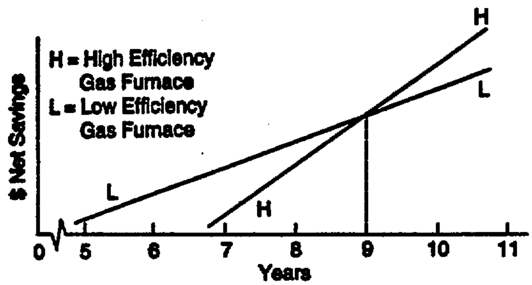

Figure 187.1 Sensitivity of net savings to holding period.

Assume that you have an old electric heating system that you are considering replacing with a gas furnace. You have a choice between a high-efficiency or low-efficiency gas furnace. You expect either to last at least 15 to 20 years. And you do not expect any significant difference in building resale value or salvage value from selecting one system over the other. So you compute the NS of each gas furnace as compared to the old electric system. You will not be able to say which system is more economical until you decide how long you will hold the building before selling it. This is where the sensitivity graph is particularly helpful.

Net savings are measured on the vertical axis, and time on the horizontal axis. The longer you hold the building, the greater will be the present value of NS from installing either of the new systems, up to the estimated life of the systems. But note what happens in the ninth year. One line crosses over another. This means that the low-efficiency system is more **cost-effective** than the high-efficiency system for any holding period up to 9 years. To the left of the crossover point, NS values are higher for the low-efficiency system than for the high-efficiency system. But for longer holding periods, the high-efficiency system is more cost-effective than the low-efficiency system. This is shown to the right of the crossover point.

How does the sensitivity graph help you decide which system to install? First, it shows that neither system is more cost-effective than the other for all holding periods. Second, it shows that the economic choice between systems is sensitive to the uncertainty of how long you hold the building. You would be economically indifferent between the two systems only if you plan to hold the building 9 years. If you plan to hold the building longer than 9 years, for example, then install the high-efficiency unit. But if you plan to hold it less than 9 years, then the low-efficiency unit is the better economic choice.

Spider Diagram for a Commercial Building Investment

Another useful graph for sensitivity analysis is the **spider diagram.** It presents a snapshot of the potential impact of uncertain input variables on project outcomes. Figure 187.2 shows—for a prospective commercial building investment—the sensitivity of the **adjusted internal rate of return** (AIRR) to three uncertain variables: project life (PL); the reinvestment rate (RR); and operation, maintenance, and replacement costs (OM&R). The spider diagram helps the investor decide if the building is likely to be a profitable investment.

Each of the three uncertain variables is represented by a labeled function that shows what AIRR value results from various values of the uncertain variable. (Although these functions are not necessarily linear, they are depicted as linear here to simplify exposition.) For example, the downward-sloping OM&R function indicates that the AIRR is inversely proportional to OM&R costs. By design, the OM&R function (as well as the other two functions) passes through the horizontal axis at the "best-guess" estimate of the AIRR (15% in this case), based on the best-

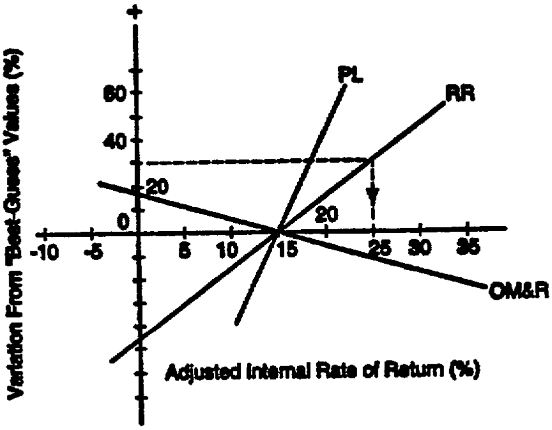

Figure 187.2 Spider diagram showing sensitivity of the adjusted internal rate of return to variations in uncertain variables. PL = project life; RR = reinvestment rate; and OM&R = operation, maintenance, and replacement costs.

guess estimates of the three uncertain variables. Other variables (e.g., occupancy rate) will impact the AIRR, but these are assumed to be known for the purpose of this analysis. Since each of the variables is measured by different units (years, percent, and money), the vertical axis is denominated in positive and negative percent changes from the best-guess values fixed at the horizontal axis. The AIRR value corresponding to any given percent variation indicated by a point on the function is found by extending a line perpendicular to the horizontal axis and directly reading the AIRR value. Thus a 30% increase in the best-guess reinvestment rate would yield a 25% AIRR, assuming that other values remain unchanged. Note that if the measure of AIRR were also given in percent differences, then the best-guess AIRR would be at the origin.

The spider diagram's contribution to decision making is its instant picture of the relative importance of several uncertain variables. In this case, the lesser the slope of a function is, the more sensitive is the AIRR to that variable. For example, any given percent change in OM&R will have a greater impact on the AIRR than will an equal percent change in RR or PL, and a percentage change in RR will have a greater impact than an equal percentage change in PL. Thus an investor will want to know as much as possible about likely OM&R costs for this project, because a relatively small variation in estimated costs could make the project a loser.

187.2 Advantages and Disadvantages

There are several advantages of using sensitivity analysis in engineering economics. First, it shows how significant any given input variable is in determining a project's economic worth. It does this by displaying the range of possible project outcomes for a range of input values, which shows decision makers the input values that would make the project a loser or a winner. Sensitivity analysis also helps identify critical inputs in order to facilitate choosing where to spend extra resources in data collection and in improving data estimates.

Second, sensitivity analysis is an excellent technique to help in anticipating and preparing for the "what if" questions that are asked in presenting and defending a project. For instance, when one is asked what the outcome will be if operating costs are 50% more expensive than expected, one will be ready with an answer. Generating answers to "what if" questions will help assess how well a proposal will stand up to scrutiny.

Third, sensitivity analysis does not require the use of probabilities, as do many techniques for treating uncertainty.

Fourth, sensitivity analysis can be used on any measure of project worth.

And, finally, sensitivity analysis can be used when there is little information, resources, and time for more sophisticated techniques.

The major disadvantage of sensitivity analysis is that there is no explicit probabilistic measure of risk exposure. That is, although one might be sure that one of several outcomes might happen, the analysis contains no explicit measure of their respective likelihoods.

Defining Terms

Adjusted internal rate of return (AIRR): The annual percentage yield from a project over the study period, taking into account the returns from reinvested receipts.

Breakeven: A combination of benefits (savings or revenues) that just offset costs, such that a project generates neither profits nor losses.

Cost-effective: The condition whereby the present value benefits (savings) of an investment alternative exceed its present value costs.

Discount rate: The minimum acceptable rate of return used in converting benefits and costs occurring at different times to their equivalent values at a common time. Discount rates reflect the investor's time value of money (or opportunity cost). "Real" discount rates reflect time value apart from changes in the purchasing power of the dollar (i.e., exclude inflation or deflation) and are used to discount constant dollar cash flows. "Nominal" or "market" discount rates include changes in the purchasing power of the dollar (i.e., include inflation or deflation) and are used to discount current dollar cash flows.

Measures of project worth: Economic methods that combine project benefits (savings) and costs in various ways to evaluate the economic value of a project. Examples are life-cycle costs, net benefits or net savings, benefit-to-cost ratio or savings-to-investment ratio, and adjusted internal rate of return.

Net savings: The difference between savings and costs, where both are discounted to present or annual values. The net savings method is used to measure project worth.

Present value: The time-equivalent value at a specified base time (the present) of past, present, and future cash flows.

Risk exposure: The probability that a project's economic outcome is different from what is desired (the target) or what is acceptable.

Sensitivity analysis: A technique for measuring the impact on project outcomes of changing one or more key input values about which there is uncertainty.

Spider diagram: A graph that compares the potential impact, taking one input at a time, of several uncertain input variables on project outcomes.

Study period: The length of time over which an investment is evaluated.

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Further Information

Uncertainty and Risk, part II in a series on least-cost energy decisions for buildings. National Institute of Standards and Technology, 1992. VHS tape and companion workbook are available from Video Transfer, Inc., 5709-B Arundel Avenue, Rockville, MD 20852. Phone: (301)881-0270.

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